Timelines with Temporal Uncertainty

Alessandro Cimatti **Andrea Micheli** Marco Roveri

Embedded Systems Unit
Fondazione Bruno Kessler, Trento, Italy
amicheli@fbk.eu

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Outline

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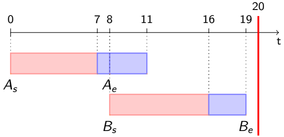
Fixed Activities (Scheduling)		

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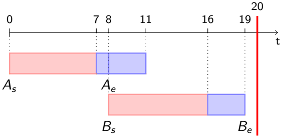
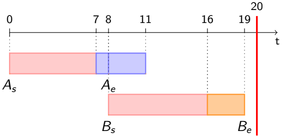
(Scheduling)</td><td>
</td><td>
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Generate a sequence of **activities** for a set of components according to a *Domain Theory* that fulfill a set of (temporal) constraints.

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Applications:
Timeline-based planning is used in many practical applications where temporal constraints are predominant (e.g. Activity Planning & Scheduling for Space Operations).

Contributions

1. Formalization of Timeline Planning with and without Temporal Uncertainty
    - Abstract syntax
    - Problem definition
    - Formal semantics

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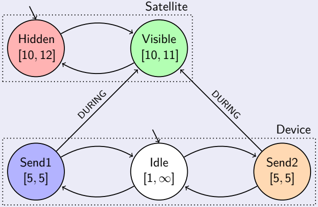
   - Abstract syntax
   - Problem definition
   - Formal semantics
2. Bounded-horizon, strong controllability problem sound and complete encoding in first-order logic.
   - Directly derived from formal semantics
   - APSI-derived concrete syntax
   - Made practical by SMT($\mathcal{LRA}$)

Outline

Formalization of Timelines (without Temporal Uncertainty)

Formalization



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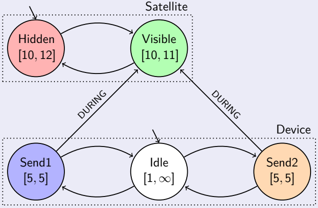

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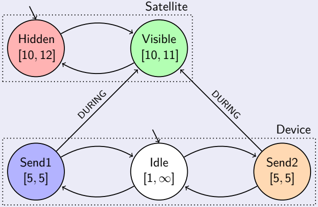

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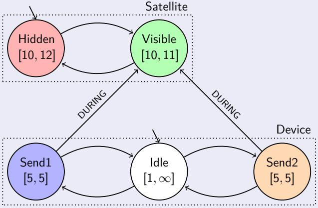

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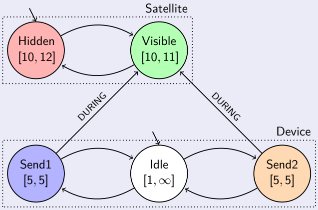

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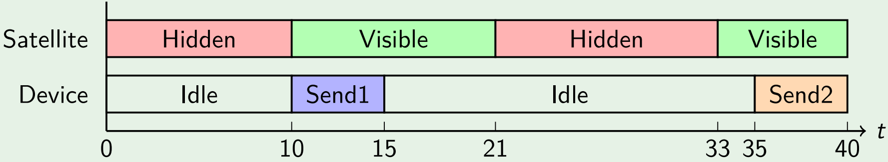

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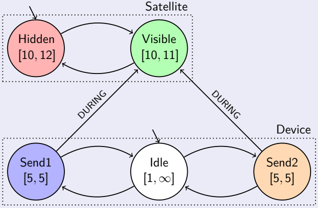

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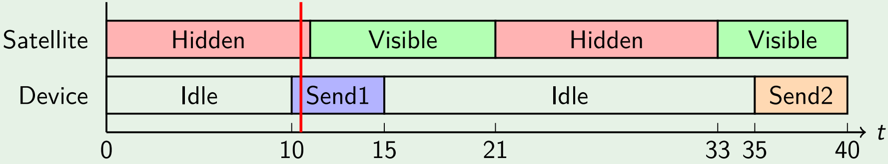

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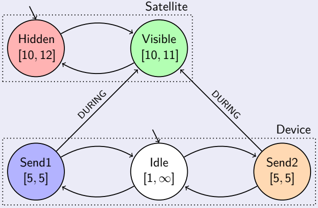

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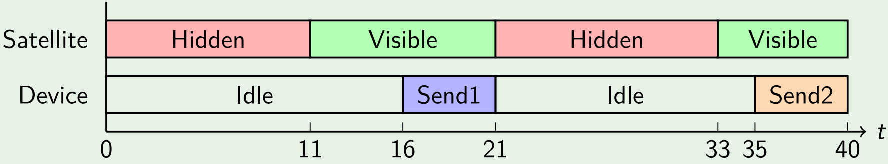

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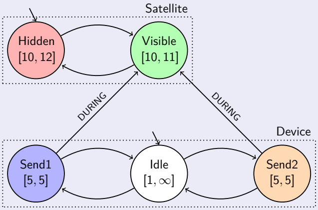

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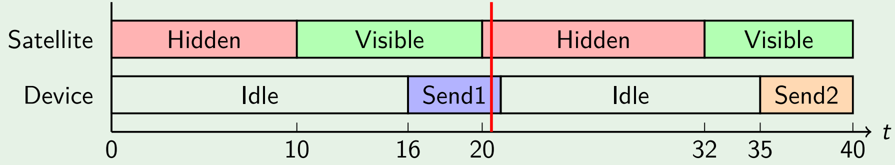
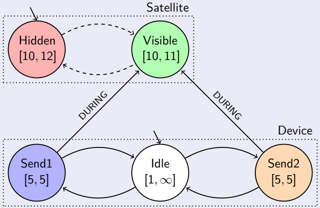


Timelines with Temporal Uncertainty

Temporal Uncertainty Annotation



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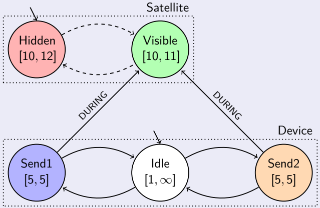

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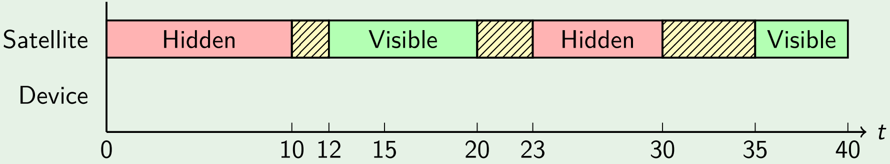

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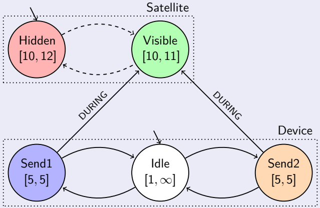

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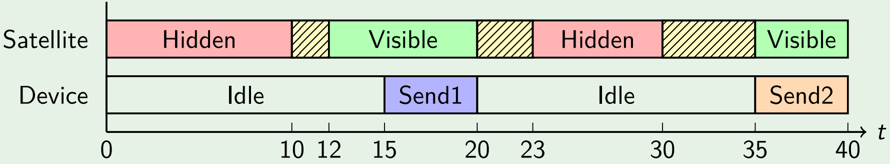

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Strong Controllability Bounded-Horizon Encoding

Idea: we assume all durations positive and fix (an upper bound of) the *maximal* number of value changes for each generator withing a given horizon.

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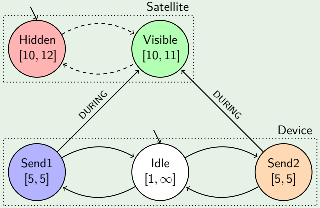

With horizon $H \doteq 240$ we have

- at most 24 values for the Satellite
- at most 80 values for the Device

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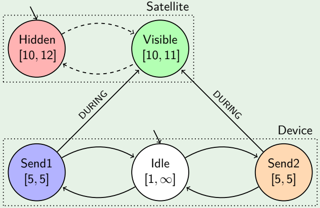

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We can **"unroll"** the problem and we encode it in (**quantified**) First Order Logic modulo the Linear Rational Arithmetic.

Experiments

SMT-Based Implementation

- Implemented on top of the NuSMV model checker
- Fourier-Motzkin Quantifier Elimination to get rid of quantifiers
- MathSAT5 to solve the SMT problems

Experimental Setup

- Three Domains with different problems
- Monolithic vs Incremental implementation
- TO is 1800s, MO is 4Gb

Type	Problem	Monolithic		Incremental	
		Time(s)	Memory(Mb)	Time(s)	Memory(Mb)
Sat	Satellite	6.87	111.5	1.88	31.9
	Machinery1	TO	TO	360.15	611.5
	Meeting	MO	MO	182.52	1897.0
Unsat	Satellite	7.17	126.2	171.25	147.6
	Machinery2	104.86	253.7	113.53	284.4
	Meeting	23.12	630.8	105.17	776.9

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Future works

- Dynamic and Weak Controllability Planning Problems
- Formalization of resources
- Optimizing Planning: find a solution that minimizes a given cost function
- Competitive implementation

Thanks

Please, come to the poster session for details, explanations and discussion!

Thanks for your attention!

Bibliography

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J. Frank and A.K. Jónsson. Constraint-based Attribute and Interval Planning. *Constraints*, 8(4): 339–364, oct 2003. ISSN 1383-7133 (Print) 1572-9354 (Online).

N. Muscettola. Hsts: Integrating planning and scheduling. Technical report, DTIC Document, 1993.

Gérard Verfaillie, Cédric Pralet, and Michel Lemaître. How to model planning and scheduling problems using constraint networks on timelines. *Knowledge Eng. Review*, 25(3):319–336, 2010.

Backup Slides

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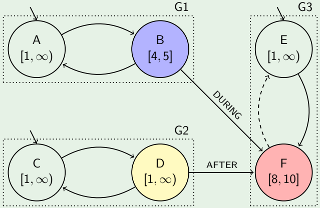
Strong Controllability Planning is not Worst Case

One may think that Strong Controllability can be solved by taking the longest or the shortest duration for an activity.

Counterexample



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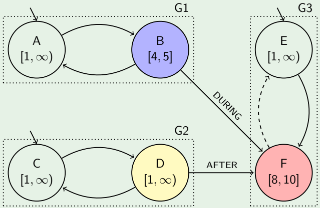

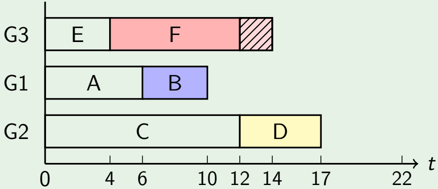

If we take the minimum duration we can violate AFTER constraint

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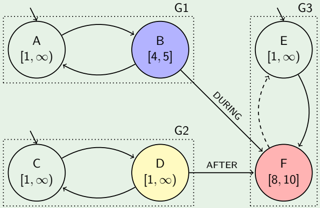

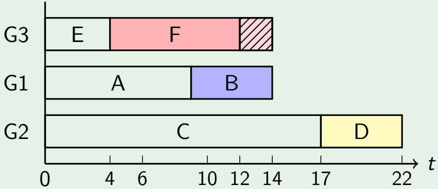

If we take the maximum duration we can violate DURING constraint

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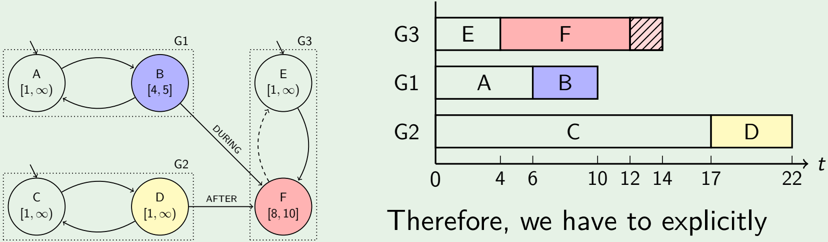
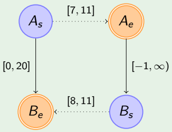


Therefore, we have to explicitly consider temporal uncertainty!

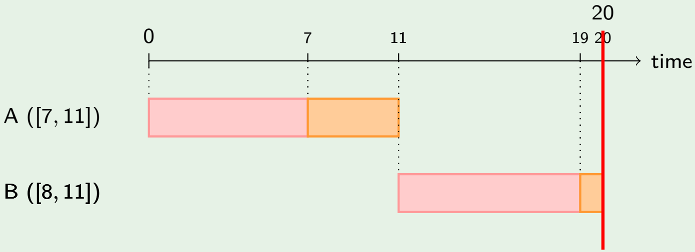
Schedules and Strategies Examples

Example



Fixed Schedule
(Strong Controllability)

- $start(A)$ at 0
- $start(B)$ at 11



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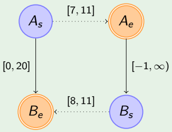
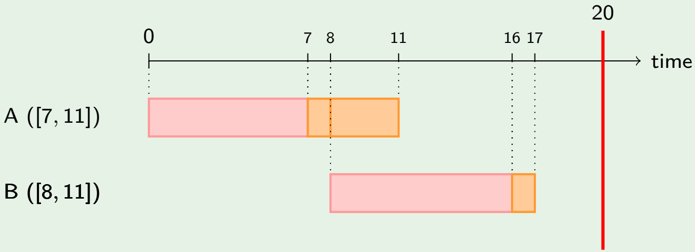


Dynamic Strategy (Dynamic Controllability)

- *start*(A) at 0
- *start*(B) at A_e



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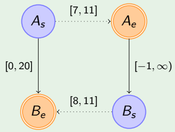
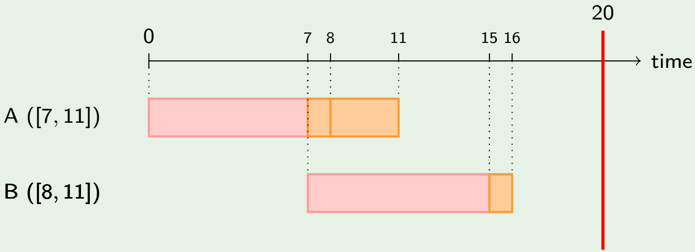


Clairvoyant Strategy (Weak Controllability)

- *start*(A) at 0
- *start*(B) at $A_e - 1$



Satisfiability Modulo Theory (*SMT*)

SMT is the problem of deciding satisfiability of a first-order Boolean combination of theory atoms in a given theory T.

Given a formula ϕ, ϕ is satisfiable if there exists a model μ such that $\mu \models \phi$.

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$\phi \doteq (\forall x.(x > 0) \vee (y \geq x)) \wedge (z \geq y)$
is satisfiable in the theory of linear real arithmetic because

$$\mu = \{(y, 6),\ (z, 8)\}$$

is a model that satisfies ϕ.

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Theories

Various theories can be used.

In this work:

- $\mathcal{LRA}$ (*Linear Real Arithmetic*)
- $\mathcal{QF\_LRA}$ (*Quantifier-Free Linear Real Arithmetic*)

Quantifier Elimination

Quantifier Elimination Definition

A theory T has quantifier elimination if for every formula Φ, there exists another formula Φ_{QF} without quantifiers which is *equivalent* to it (modulo the theory T)

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Quantifier Elimination for $\mathcal{LRA}$

$\mathcal{LRA}$ theory admits quantifier elimination, but elimination algorithms are very costly (doubly exponential in the size of the original formula).

$(\exists x.(x \geq 2y + z) \wedge (x \leq 3z + 5)) \leftrightarrow (2y - 2z - 5 \leq 0)$

Different techniques exists:

- Fourier-Motzkin
- Loos-Weisspfenning
- ...

Quantifier Elimination for $\mathcal{LRA}$

Various techniques

- Fourier-Motzkin
- Loos-Weisspfenning
- ...

Fourier-Motzkin Elimination

- Procedure that eliminates a variable from a **conjunction** of linear inequalities.
- It can be applied to a general $\mathcal{LRA}$ formula by computing the DNF and applying the technique to each disjunct.
- The complexity is doubly exponential: in the number of variable to quantify and in the size of the DNF formula.

Fourier-Motzkin Elimination

Let $\psi \doteq \exists x_r . \bigwedge_{i=0}^{N} \sum_{k=1}^{M} a_{ik} x_k \leq b_i$ be the problem we want to solve, where x_r is the variable to eliminate.

We have three kinds of inequalities in a system of linear inequalities:

- $x_r \geq A_h$, where $A_h \doteq b_i - \sum_{k=1}^{r_i-1} a_{ik} x_k$, for $h \in [1, H_A]$
- $x_r \leq B_h$, where $B_h \doteq b_i - \sum_{k=1}^{r_i-1} a_{ik} x_k$, for $h \in [1, H_B]$
- Inequalities in which x_r has no role. Let ϕ be the conjunction of those inequalities.

The system is **equivalent** to $(max_{h=1}^{H_A}(A_h) \leq x_r \leq min_{h=1}^{H_b}(B_h)) \wedge \phi$ and to $(max_{h=1}^{H_A}(A_h) \leq min_{h=1}^{H_b}(B_h)) \wedge \phi$

max and *min* are not linear functions, but we can mimic the formula by using a quadratic number of linear inequalities:

$$\psi \Leftrightarrow (\bigwedge_{i=0}^{H_A} \bigwedge_{j=0}^{H_B} A_i \leq B_j) \wedge \phi$$

Fourier-Motzkin Example

Fourier Motzkin Example: Step 1

Let $\psi \doteq \forall z.((z \geq 4) \rightarrow ((x < z) \wedge (y < z)))$.

We convert all the quantifiers in existentials and we compute the DNF of the quantified part of the formula.

$\psi \Leftrightarrow \neg \exists z.((z \geq 4) \wedge \neg((x < z) \wedge (y < z)))$

$\psi \Leftrightarrow \neg \exists z.((z \geq 4) \wedge (\neg(x < z) \vee \neg(y < z)))$

$\psi \Leftrightarrow \neg \exists z.(((z \geq 4) \wedge \neg(x < z)) \vee ((z \geq 4) \wedge \neg(y < z)))$

Fourier Motzkin Example: Step 2

For every disjunct, we apply the Fourier-Motzkin Elimination:

$((z \geq 4) \wedge (z \leq x)) \Leftrightarrow (4 \leq x)$

$((z \geq 4) \wedge (z \leq y)) \Leftrightarrow (4 \leq y)$

Then, we rebuild the formula:

$\psi \Leftrightarrow \neg((4 \leq x) \vee (4 \leq y))$

$\psi \Leftrightarrow ((x < 4) \wedge (y < 4))$

Temporal Uncertainty Characterization

Temporal Uncertainty can be seen as a **game** between an *Executor* and the adversarial *Nature*.

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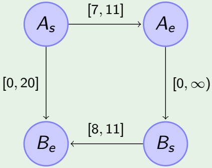
Temporal Problems (with Temporal Uncertainty)

Temporal Problems



A_s, A_e, B_s, B_e are **Time Points (X_c)**

$\longrightarrow$ represents **Free Constraints (C_f)**

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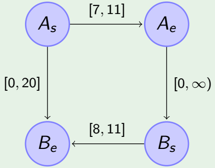

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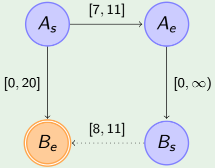
Temporal Problems with Uncertainty



A_s, A_e, B_s are **Controllable Time Points (X_c)**
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Controllability Levels

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- **Strong Controllability (No observation)**
  Find a **fixed schedule** for controllable time points

Fixed Schedule

- $start(A)$ at 0
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  Find a **strategy that depends on past observations only**, for scheduling controllable time points

Fixed Schedule

- $start(A)$ at 0
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Dynamic Strategy

- $start(A)$ at 0
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Controllability Levels

- **Strong Controllability (No observation)**
  Find a **fixed schedule** for controllable time points

- **Dynamic Controllability (Past observation)**
  Find a **strategy that depends on past observations only**, for scheduling controllable time points

- **Weak Controllability (Full observation)**
  Find a **"clairvoyant" strategy** for scheduling controllable time points

Fixed Schedule

- $start(A)$ at 0
- $start(B)$ at 11

Dynamic Strategy

- $start(A)$ at 0
- $start(B)$ at C

Clairvoyant Strategy

- $start(A)$ at 0
- $start(B)$ at $C - 1$