Compiling Away Uncertainty in Strong Temporal Planning with Uncontrollable Durations

Andrea Micheli
FBK and University of Trento
Trento, 38123, Italy
amicheli@fbk.eu

Minh Do and David E. Smith
NASA Ames Research Center
Moffet Field, CA, 94035, USA
{minh.do, david.smith}@nasa.gov

Abstract
Real world temporal planning often involves dealing with uncertainty about the duration of actions. In this paper, we describe a sound-and-complete compilation technique for strong planning that reduces any planning instance with uncertainty in the duration of actions to a plain temporal planning problem without uncertainty.

We evaluate our technique by comparing it with a recently-presented technique for PDDL domains with temporal uncertainty. The experimental results demonstrate the practical applicability of our approach and show a complementary behavior with respect to the previous techniques. We also demonstrate the high expressiveness of the translation by applying it to a significant fragment of the ANML language.

1 Introduction
For most real world planning problems there is uncertainty about the duration of actions. For example, robots and rovers have transit times that are highly uncertain due to terrain, obstacle avoidance, and traction. There is also uncertainty in the duration of manipulation and communication tasks. For cars and trucks, transit times are uncertain due to traffic, road conditions and traffic signals. Any actions to be executed by humans are also likely to have uncertain durations. While there are domains in which the variability on the action duration is small enough that it can be ignored, there are many others where it can be significant, such as transit times during rush hour.

When there are no time constraints, no required concurrency, and plan duration is unimportant, this uncertainty can often be ignored to find a feasible plan. However, if there are exogenous events that affect action conditions, or time-constrained goals, the uncertainty on action durations must be considered.

In general, temporal conditional planning is very hard, particularly for actions with duration uncertainty (Younes and Simmons 2004; Mausam and Weld 2008; Beaudry, Kabanza, and Michaud 2010). In practice, most practical planners take one of two much simpler approaches:

1. Plan using expected action durations, and rely on runtime replanning and plan flexibility to deal with actions that take more or less time than expected.

2. Plan using worst case action durations.

The first of these approaches is risky – there is no guarantee that the plan will succeed or that runtime replanning can achieve the goals. The second approach, while generally more conservative, can also fail if there are time constraints or goals with lower bounds (e.g. an action should not be completed, or a goal should not be completed before some particular time). For space applications where any failure during plan execution is potentially very costly, having a plan that is guaranteed to execute successfully is often critical.

Recently, Cimatti, Micheli, and Roveri (2015) addressed these issues by extending PDDL2.1 to explicitly model duration range for actions, and devised a planner that soundly reasons to produce robust plans. In that work, the authors introduced the “Strong Planning Problem with Temporal Uncertainty” (SPPTU) as the problem of finding a sequence of action instances and fixed starting times, such that for every possible duration of each action in the plan, the plan is valid and leads to the goal. In this work, we address the same problem. However, to make it more relevant to real-world applications, we consider a much richer language for representing temporal planning domains. Specifically, we use and support: (i) a variable-value language; (ii) durative conditions over arbitrary sub-intervals of actions; (iii) effects at arbitrary time points during an action, (iv) exogenous events; (v) disjunctive conditions; and (vi) temporal constraints on goals. We address the SPPTU by automatically translating a planning instance with uncertainty on action durations into a plain temporal planning problem with controllable action durations.

We also present two sets of experimental evaluations of the compilation technique showing that it can be practically applied on expressive domains:

- On existing PDDL2.1-extended benchmarks: comparing against the techniques proposed in (Cimatti, Micheli, and Roveri 2015)
- On a set of problems modeled in ANML (Smith, Frank, and Cushing 2008), which enables modeling realistic tem-
poral planning domains more naturally.

2 Related Work

Temporal uncertainty is a well-understood concept in scheduling and has been widely studied (Morris 2006; e Assis Santana and Williams 2012; Mui, Beck, and McIlraith 2013; Cimatti, Micheli, and Roveri 2014). The problem we address can be seen as a generalization of Strong Controllability for Temporal Problems (Vidal and Fargier 1999; Peintner, Venable, and Yorke-Smith 2007) to planning rather than scheduling. Dealing with planning is much harder because the actions (and thus the time points associated with them) are not known a-priori and must be searched for. Moreover causal relationships between actions are much more complex.

In temporal planning, duration uncertainty is a known challenge (Bresina et al. 2002), but few temporal planners address it explicitly. Some temporal planners (Frank and Jönsson 2003; Cesta et al. 2009) cope with this issue by generating flexible temporal plans: instead of fixing the execution time of each action, they return a (compactly represented) set of plans that must be scheduled at run-time by the plan executor. This approach cannot guarantee plan executability and goal achievement at runtime, because there is no formal modeling of the boundaries and contingencies in which the system is going to operate. In addition, this requires that the executor be able to schedule activities at runtime. In fact, flexibility and controllability are complementary: controllability provides guarantees with respect to the modeled uncertainty, while flexibility allows the plan to be adjusted during execution. In principle, we can use any temporal planner that can generate flexible plans (e.g., VHPOP) in combination with our compilation to generate a flexible strong plan.

IxTeT (Ghallab and Laruelle 1994) was the first attempt to apply the results in temporal reasoning under uncertainty to planning, but the planner demanded the scheduling of a Simple Temporal Network with Uncertainty (STNU) (Vidal and Fargier 1999) by the plan executor. Here, we aim at generating plans that are guaranteed to work regardless of the temporal uncertainty. Nonetheless, IxTeT provides dynamic controllability: it generates a strategy for scheduling the actions depending on contingent observations. Although dynamic plans indeed can work in more situations than strong plans, they are also complex to generate, understand, and execute. When safety is paramount (e.g., space applications), dynamic plans might not be permitted because they require run-time computation that is hard to certify and to execute on-board in real-time. Strong plans are simple to execute and to check, and are suitable for safety critical systems where guarantees are needed for the uncertainty and sub-optimality is an acceptable price to pay.

In contrast to Beaudry, Kabanza, and Michaud (2010) we are concerned with qualitative uncertainty, meaning that we are not dealing with probability distributions, but only with durations that are bounded in convex intervals. In addition, we aim to guarantee goal achievement, while Beaudry, Kabanza, and Michaud maximize the probabilistic expectation.

There is a clear parallel between the problem we are solving and conformant planning (Ghallab, Nau, and Traverso 2004). In this sense, our work is similar to (Palacios and Geffner 2009) in which the authors transform conformant planning into deterministic planning, although the translation is very different.

The closest work to ours is that of Cimatti, Micheli, and Roveri (2013, 2015). Cimatti, Micheli, and Roveri (2013) present a logical characterization of the SPPTU for timelines with temporal uncertainty, as well as a first-order encoding of the problem having bounded horizon. Cimatti, Micheli, and Roveri (2015) cast the same idea in PDDL by extending state-space temporal planning. In this paper, we generalize both these frameworks, as we do not impose any bounded horizon for the problem and we consider a more expressive language allowing disjunctive preconditions, effects at arbitrary time points during actions and durative conditions on arbitrary sub-intervals. In Section 5 we provide a comparison with the techniques proposed in (Cimatti, Micheli, and Roveri 2015).

3 Modeling Duration Uncertainty

In (Cimatti, Micheli, and Roveri 2015), the authors propose an extension of PDDL 2.1 to model actions with uncontrollable duration. In this paper we use a richer language that includes timed-initial-literals (PDDL 2.2), durative goals (PDDL 3.0), and multi-valued variables (PDDL 3.1). In addition, we extend the language to allow conditions expressed over sub-intervals of actions, and effects at arbitrary time points during an action. These features turn out to be particularly useful for encoding many problems of interest, and for encoding our translation. To simplify the presentation, we exclude some features of PDDL that are orthogonal to our approach of handling temporal uncertainty, such as numeric variables and domain axioms. Our techniques will work whether or not those features are included.
at-end condition holding at the \( st_p \) time point. When \( st_p = st_a \) and \( et_p = et_a \), then \( p \) is an overall durative condition holding in the open interval \((st_p, et_p)\), \( f \in V \) is a proposition with value \( v = T \) or \( v = F \) over the specified time period.

- \( E_a \) is a set of instantaneous effects, each \( e \in E_a \) is of the form \([t_e, f := v] \) where \( t_e = st_a \) or \( t_e = et_a \) is the time at which the at-start or at-end effect \( e \) occurs.

We allow disjunctive action conditions \( p \) of the form \((st_p, et_p) \sqcup_{i=1}^n f_i = v_i \) in which \( p \) is satisfied if at least one disjunct is satisfied for every time point in \((st_p, et_p)\).

A plan \( \pi \) of \( P \) is a set of tuples \( \langle t_a, a \rangle \), in which actions \( a \in A \) are associated to wall-clock start times \( t_a \). \( \pi \) is valid if it is executable in \( I \) and achieves all goals in \( G \).

We extend the above features of PDDL 2.2 to include the following features from PDDL 3.0 and 3.1:

- Multi-valued variables: introduced in PDDL 3.1, allow variables \( f \) in \( V \) to have domains \( Dom(f) \) with arbitrary values, instead of just \( T \) and \( F \).

- Durative goals: which can be modeled as constraints in PDDL 3.0, allow each goal \( g \in G \) to be associated with an interval \([st_g, et_g] \) specifying when the goal must be true. In this setting, we allow the time constant \( et_a \) that indicates that the goal must be reached at the end of the plan.

Beyond PDDL. Additionally, the key features in our framework that go beyond PDDL are: (1) actions can have uncontrollable durations, and (2) action conditions and effects are not restricted to action start or end time points. Specifically:

1. Action duration \( d_a \) is replaced by an interval \([d_{a^l}, d_{a^u}] \) specifying lower- and upper-bound values on action duration: \( d_{a^l} \leq d_a \leq d_{a^u} \). We further divide the set of actions \( A \) into two subsets:
   - **Controllable** actions \( A_c \), where the duration can be chosen by the planner within the bounds \([d_{a^l}, d_{a^u}] \).
   - **Uncontrollable** actions \( A_u \), where the duration is not under the planner’s control.

2. Instead of constraining the times \( st_p \) and \( et_p \) of each condition \( p \) or the time \( t_e \) of effect \( e \) to be either \( st_a \) or \( et_a \), we allow each of them to take an arbitrary value: \( st_a + \delta \) or \( et_a - \delta \), with \( \delta \in \mathbb{R}^+ \) (the temporal constraint \( st_p \leq et_p \) should still be satisfied). We require \( \delta \) to be positive and less than or equal to the action minimal duration to prevent effects before the start or after the end of the action. Analogously to PDDL, If \( st_p = et_p \) the condition is instantaneous and is required to hold at the single point \( st_p \); otherwise, the condition is required to hold in the open interval \((st_p, et_p)\).

A (strong) plan \( \pi^u \) for a planning problem with uncertainty \( P^u \) is valid iff each instance of \( \pi^u \), obtained by fixing the duration \( d_a \) for each uncontrollable action \( a \in \pi^u \) to any value within \([d_{a^l}, d_{a^u}] \), is a valid plan.

**Example.** A rover, initially at location \( l_1 \), needs to transmit some science data from location \( l_2 \) to an orbiter that is only visible in the time window \([14, 30] \). The rover can move from \( l_1 \) to \( l_2 \) using an action \( move \) (abbreviated \( \mu \)) that has uncontrollable duration between 10 and 15 time units. The data transmission action \( transmit \) (abbreviated \( \tau \)) takes between 5 and 8 time units to complete. The goal of the rover is to transmit the data to the orbiter. Because of the harsh day-time temperatures at location \( l_2 \) the rover cannot arrive at \( l_2 \) until the sun goes behind the mountains at time 15. Figure 1 illustrates this scenario, which we encode as:

\[
\begin{align*}
V &\doteq \{ pos : \langle l_1, l_2 \rangle, visible : \langle T, F \rangle, hot : \langle T, F \rangle, sent : \langle T, F \rangle \} \\
I &\doteq \{ pos = l_1, visible = F, sent = F, hot = T \} \\
T &\doteq \{ \langle 14 \rangle visible := T, \langle 30 \rangle visible := F, \langle 15 \rangle hot := F \} \\
G &\doteq \{ \langle et_{\mu}, et_{\tau} \rangle sent = T \} \\
A_c &\doteq \emptyset \\
A_u &\doteq \{ \langle 10, 15 \rangle, C_{\mu}, E_{\mu} \}, \{ \langle 5, 8 \rangle, C_{\tau}, E_{\tau} \} \\
C_{\mu} &\doteq \{ \langle et_{\mu}, st_{\mu} \rangle pos = l_1 \} \cup \{ et_{\mu}, et_{\mu} \ hot = F \} \\
C_{\tau} &\doteq \{ \langle st_{\tau}, et_{\tau} \rangle pos = l_2 \} \cup \{ st_{\tau}, et_{\tau} \ visible = T \} \\
E_{\mu} &\doteq \{ \langle et_{\mu} \rangle pos := l_2 \} \\
E_{\tau} &\doteq \{ \langle et_{\tau} \rangle sent = T \} 
\end{align*}
\]

Figure 2 graphically shows a valid plan:

\[ \pi^u \doteq \{ \langle 6, move(l_1 \rightarrow l_2) \rangle, \langle 22, transmit \rangle \} \]

Note that all the actions in \( \pi^u \) have uncontrollable duration.

**Discussion.** In general, finding a strong plan for a problem with duration uncertainty is different from simply considering the maximum or the minimum duration for each action. Consider our rover example and its strong plan shown in Figure 2. The \( \mu \) (i.e., \( move \)) action must terminate before the transmit action can start and, at the same time, \( \mu \) cannot terminate before time 15 due to the temperature constraint. If we only consider the lower-bound on the duration of \( \mu \) (i.e., planning with \( d_{\mu}^l = 10 \)) then one valid plan is: \( \pi_{lb} \doteq \{ \langle 11, \mu \rangle, \langle 22, \tau \rangle \} \). However, because of the uncertainty in the actual execution duration of \( \mu \), it may actually take 14 time units to arrive at \( l_2 \). Thus, the rover would start transmitting at time 22 before it actually arrives at \( l_2 \) at time 11 + 14 = 25. Thus, plan \( \pi_{lb} \) is invalid. Similarly, if we consider only the maximal duration (i.e., planning with \( d_{\mu}^{ub} = 15 \)), then one possible plan would
Disjunctive Conditions: In contrast to ordinary temporal planning, it is not possible to compile away disjunctive conditions using the action duplication technique (Gazen and Knoblock 1997), because the set of satisfied disjuncts in the presence of uncertainty can depend on the continuation. For example, consider an action $a$ starting at time $t$ where two Boolean variables $p_1$ and $p_2$ are $F$. $a$ has uncontrollable duration in $[l, u]$, an at-start effect $e_1 = \langle [st_a] p_1 := T \rangle$ and two at-end effects $e_2 = \langle [et_a] p_1 := F \rangle$ and $e_3 = \langle [et_a] p_2 := T \rangle$. An at-start condition $p_1 \lor p_2$ of another action $b$ is satisfied anywhere between the start of the action $a$ and the next deletion of $p_2$. Thus, $b$ can start anytime within $d = [t + l, t + u]$. However, if we compile away this disjunctive condition by replacing $b$ with two actions $b_1$ and $b_2$: one with an at-start condition $p_1$ and the other with an at-start condition $p_2$, then $b_1$ is not executable within $d$ because there is no time point in $d$ in which we can guarantee that $p_1 = T$ (because $a$ may take the minimum duration $l$ and thus the at-end effect $e_2$ will occur at $t + l$ to set $p_1 = F$). Similarly, we cannot start $b_2$ within $d$ because we also cannot guarantee that $p_2 = T$ at any time point within $d$ (because $a$ may take the maximum duration $u$ and thus $e_3$ that set $p_2 = T$ will not happen until $t + u$). Thus, compiling away disjunctive conditions leads to incompleteness when there are uncontrollable durative actions. For this reason it is important to explicitly model disjunctive conditions in our language.

4 Compilation Technique

In this section, we present our compilation technique, which can be used to reduce any planning instance $P$ having duration uncertainty into a temporal planning instance $P'$ in which all actions have controllable durations. The translation guarantees that $P$ is solvable if and only if $P'$ is solvable. Moreover, given any plan for $P'$ we can derive a plan for $P$. This approach comes at the cost of duplicating some of the variables in the domain, but allows for the use of off-the-shelf temporal planners.

The overall intuition behind the translation is the following. Consider the $\text{transmit}$ (i.e., $\tau$) action in our example, and suppose it is scheduled to start at time $k$. Let $v$ be the value of $\text{sent}$ at time $k + 5$; since $\text{transmit}$ has an at-end effect $\langle [et_a] \text{sent} := T \rangle$, we know that the value of the variable $\text{sent}$ during the interval $(k + 5, k + 8]$ will be either $v$ or $T$ depending on the duration of the action. After time $k + 8$ we are sure that the effect took place, and we are sure of the value of $\text{sent}$ until another effect is applied. Since we are not allowed to observe anything at run-time in strong planning, we need to consider this uncertainty in the value of $\text{sent}$ and produce a plan that works regardless. Since $\text{sent}$ could appear as a condition of another action (or as a goal condition, as in our example) we must rewrite such conditions to be true only if both $T$ and $v$ are values that satisfy the condition.

To achieve this, we create an additional variable $\text{sent}_\sigma$ (the shadow variable of $\text{sent}$). This secondary variable stores the alternative value of $\text{sent}$ during uncertainty periods. When there is no uncertainty in the value of $\text{sent}$, both $\text{sent}$ and $\text{sent}_\sigma$ will have the same value. In this way, all the conditions involving $\text{sent}$ can be rewritten in terms of $\text{sent}$ and $\text{sent}_\sigma$ to ensure they are satisfied by both the values.

In general, our translation works by rewriting a planning instance $P = \langle V, I, T, G, A \rangle$ into a new planning instance $P' = \langle V', I', T', G', A' \rangle$ that does not contain actions with uncontrollable duration.

Uncertain Variables. The first step is to identify the set of variables $L \subseteq V$ that appear as effects of uncontrollable actions and are executed at a time depending on the end of the action.

$$L = \{ f \mid a \in A_u, \langle [t] f := v \rangle \in E_u, t = et_a - \delta \}$$

Intuitively, this is the set of variables that can possibly have uncertain value during plan execution. A variable that is modified only at times linked to the start of actions or by timed initial literals, cannot be uncertain as neither the starting time of actions nor the timed initial literals can be uncertain in our model. In our running example, the set $L$ becomes $\{ \text{sent, pos} \}$

We now define the set $V'$ as the original variables $V$ plus a shadow variable for each variable appearing in $L$.

$$V' = V \cup \{ f_\sigma \mid f \in L \}$$

We use the pair of variables $f$ and $f_\sigma$ to represent uncertainty: if $f = f_\sigma$ we know that there is no uncertainty in the value of $f$, while if $f \neq f_\sigma$ we know that the actual value of $f$ in the original problem is either $f$ or $f_\sigma$.

Disjunctive Conditions. At the end of Section 3, we outlined the reason why existing approaches of compiling away disjunctive conditions will not work with uncontrollable action durations. In order to rewrite a disjunctive condition $p = \langle [st_p, et_p] \bigvee_{i=1}^n f_i = v_i \rangle$ we need to ensure that the

\[\text{hot = T, visible = F, move, transmit, visible = T, visible = F}\]

Figure 2: Graphical execution of $\pi^u$. Striped regions represent the uncertainty in the action duration.

\[\text{6, 14, 15, 16, 21, 22, 27, 30}\]
result is satisfied if and only if both the values of \( f \) and \( f_a \) for each \( f \in L \) are satisfying values for \( p \). For this reason, we define an auxiliary function \( \chi(\psi) \) that takes a single disjunctive condition in \( P \) and returns a set of disjunctive conditions in \( P' \).

\[
\chi(\psi) = \begin{cases} 
\{(f = v)\} & \text{if } \psi \equiv (f = v), f \not\in L \\
\{(f = v), (f_a = v)\} & \text{if } \psi \equiv (f = v), f \in L \\
\{r \lor s \mid r \in \chi(\psi_1), s \in \chi(\psi_2)\} & \text{if } \psi \equiv \psi_1 \lor \psi_2
\end{cases}
\]

For example, the condition of the \( \tau \) action, \( \text{pos} = l_2 \), is translated as the two conditions \( \text{pos} = l_2 \) and \( \text{pos}_\sigma = l_2 \). Analogously, assuming that both \( f \) and \( g \) are in \( L \), a given condition \((f = T) \lor (g = F)\) in \( P \) is translated by function \( \chi \) as the set of conditions \( \{(f = T) \lor (g = T) \lor (g = F), (f = T) \lor (g_a = F), (f_a = T) \lor (g_a = F)\} \) in \( P' \).

**Uncertain Temporal Intervals.** We also need to identify the temporal interval in which the value of a given variable can be uncertain. Given an action \( a \) with uncertain duration \( d_a \) in \([l, u]\), let \( \lambda(t) \) and \( \nu(t) \) be the earliest and latest possible times at which an at-end effect at \( t \) may happen. Thus: \( \lambda(t) = st_a + l - \delta \) and \( \nu(t) = st_a + u + \delta \). Both functions are equal to \( t \) if \( t = st_a + \delta \). For example, the effect \( e_1 \equiv (\text{sent} := T) \) of action \( \tau \). We know that the duration of transmit is uncertain in \([5, 8]\), therefore the effect can be applied between \( \lambda(e_t) = st + 5 \) and \( \nu(e_t) = st + 8 \) and the sent variable has an uncertain value within that interval.

**Uncontrollable Actions.** For each uncontrollable action \( a = (\llbracket l, u \rrbracket, C_a, E_a) \) in \( A_u \) in the original model we create a new action \( a' = (\llbracket u, u \rrbracket, C_{a'}, E_{a'}) \) in \( A'_u \). Specifically, we first fix the maximal duration \( u \) as the only allowed duration for \( a' \) and then insert appropriate effects and conditions during the action to capture the uncertainty.

The effects \( E_{a'} \) are partitioned in two sets \( E_{a'}^l \) and \( E_{a'}^u \) to capture possible values during the uncertain action duration. The conditions \( C_{a'} \) are also composed of two elements: the rewritten conditions \( C_{a'}^R \) and the conditions added to protect the new effects \( C_{a'}^E \) (thus \( C_{a'} = C_{a'}^R \cup C_{a'}^E \)).

**Rewritten conditions \( C_{a'}^R \):** are obtained by rewriting existing action conditions by means of the \( \chi \) function. The intervals specifying the duration of the conditions are preserved; since the duration action is set to its maximum, the intervals of the conditions are “stretched” to match their maximal duration.

\[
C_{a'}^R = \{(\lambda(t_1), \nu(t_2)) \alpha \mid \alpha \in \chi(\psi), (t_1, t_2) \psi \in C_a\}
\]

For example, the set \( C_{a'}^R \) for the \( \tau \) action is: \( \{(st_a, st + 8) \text{ pos} = l_2, (st_r, st_r + 8) \text{ pos}_\sigma = l_2, (st_r, st_r + 8) \text{ visible} = T)\}. \) This requires variables \( \text{visible}, \text{pos} \) and \( \text{pos}_\sigma \) to be true throughout the execution of \( \tau \).

**Compiling action effects:** The effects of the original action are duplicated: both the affected variable \( f \) and its shadow \( f_a \) are modified, but at different times. We first identify the earliest and latest possible times at which an effect can happen due to the duration uncertainty (see earlier discussion on \( \lambda(t) \) and \( \nu(t) \)). We then apply the effect on \( f_a \) at the earliest time point \( \lambda(t) \), and at the latest time point \( \nu(t) \) we re-align

\[
\begin{array}{c}
\text{at } J_2, \text{ visible} \\
\text{transmit} \\
\text{at } J_2, \text{ visible}, \text{ at } J_{3_a} \\
\text{sent} \leftarrow T \\
\text{transmit'} \\
\text{sent}_a \leftarrow T \\
\text{sent} \leftarrow T
\end{array}
\]

Figure 3: Graphical view of the original transmit action instance (top) and its compilation (bottom).

\( f \) and \( f_a \) by also applying the effect on \( f \):

\[
E_{a'}^l = \{(\{\lambda(t)\} f_a := v) \mid \{[t] f := v\} \in E_a\}
\]

\[
E_{a'}^u = \{(\nu(t) f := v) \mid \{[t] f := v\} \in E_a\}
\]

For example, the \( \tau \) action has \( E_{a'}^l = \{(\{st_r + 5\} \text{ sent}_a := T)\} \) and \( E_{a'}^u = \{(\{st_r + 8\} \text{ sent}_a := T)\}. \)

**Additional conditions \( C_{a'}^E \):** let \( t = \text{sent}_a - \delta \) be the time of an at-end effect that affects the value of \( f \). In order to prevent other actions from changing the value of \( f \) during the interval \( \{\lambda(t), \nu(t)\} \) where the value of \( f \) is uncertain, we add a condition in \( C_{a'}^E \) to maintain the value of \( f_a \) throughout the uncertain duration \( \{\lambda(t), \nu(t)\} \).

\[
C_a' = \{(\lambda(t), \nu(t)) f_a = v \mid \{[t] f := v\} \in E_a\} \cup \{(\nu(t), \nu(t)) f_a = v \mid \{[t] f := v\} \in E_a\}
\]

We are in fact using a left-open interval \( \{\lambda(t), \nu(t)\} \) by specifying the same condition on the open interval \( \{\lambda(t), \nu(t)\} \) and the single point \( [\nu(t)] \). Since the effect on \( f_a \) (belonging to \( E_{a'}^l \)) is applied at time \( \lambda(t) \), the condition is satisfied immediately after the effect and we want to avoid concurrent modifications of either \( f \) or \( f_a \) until the uncertainty interval ends at \( \nu(t) \). For example, the \( \tau \) action has \( C_{a'}^E = \{(st_r + 5, st_r + 8) \text{ sent}_a := T)\}. \) Compilation of the \( \tau \) action is depicted in Figure 3.

**Controllable actions:** are much simpler. For each \( a = (\llbracket l, u \rrbracket, C_a, E_a) \in A \), we introduce a replacement action \( a' = (\llbracket u, u \rrbracket, C_{a'}, E_{a'}) \in A'_u \), in which: (1) each condition in \( C \) is rewritten to check the values of both the variables and their shadows, and (2) each effect is applied to a variable and its shadow, if any.

\[
C_{a'} = \{(\lambda(t_1), \nu(t_2)) \alpha \mid \alpha \in \chi(\psi), (t_1, t_2) \psi \in C_a\}
\]

\[
E_a' = E_a \cup \{(\{[t] f_a := v\} \mid f \in L, \{[t] f := v\} \in E_a\}
\]

**Initial state \( I \):** is handled by initializing variables and their corresponding shadow variables in the same way as in the original problem.

\[
I' = I \cup \{f_a = v \mid f \in L, f = v \in I\}
\]

For example, the initial state of our running problem is the original initial state plus \( \{\text{sent}_a = F, \text{pos}_\sigma = l_1\} \).

**Timed Initial Literals:** \( T' \) are set similarly to the effects.

\[
T' = T \cup \{\{[t] f_a := v\} \mid f \in L, \{[t] f := v\} \in T\}
\]
In our example, we do not have timed initial literals operating on uncertain variables, thus \( T = T' \).

**Goal conditions:** \( G \) is augmented to consider both the original and shadow variables, without modifying the application times, since they are fixed and cannot be uncertain.

\[
G' = G \cup \{ ([t_1, t_2] f_\sigma = v) \mid f \in L, ([t_1, t_2] f = v) \in G \}
\]

In our example, the set \( G' \) becomes \( \{ ([\sigma, \tau] \text{sent} = T), ([\sigma, \tau] \text{sent} = T') \} \).

**Example.** The compilation for our example problem is:

\[
V' = V \cup \{ \mu_\sigma : \{ t_1, t_2 \}, \text{sent}_\sigma : \{ T, F \} \}
\]

\[
T' = \{ ([14] \text{visible} := T), ([30] \text{visible} := F), ([15] \text{hot} := F) \}
\]

\[
G' = \{ ([\sigma, \tau] \text{sent} = T), ([\sigma, \tau] \text{sent} = T') \}
\]

\[
A'_\sigma = \{ ([15, 15], C_{\mu'}, E_{\mu'}), ([8, 8], C_{\tau'}, E_{\tau'}) \}
\]

\[
C_{\mu'} = \{ ([\sigma, \mu] \text{pos} = t_1), ([\sigma, \mu] \text{pos} = t_2), ([\sigma, \mu] \text{hot} = F), ([\sigma, \mu] + 10, \sigma, \mu + 10) \text{pos} = l_2 \}
\]

\[
C_{\tau'} = \{ ([\sigma, \tau] \text{pos} = t_2), ([\sigma, \tau] \text{pos} = t_2) \}
\]

\[
E_{\mu'} = \{ ([\sigma, \mu] + 10) \text{pos} = l_2 \}
\]

\[
E_{\tau'} = \{ ([\sigma, \tau] + 5) \text{sent} = T \}
\]

**Discussion.** This compilation is sound and complete. Thus, the original problem is solvable if and only if the resulting problem is solvable. Any plan for the rewritten temporal planning problem is automatically a strong plan for the original problem (with the obvious mapping from the rewritten to the original actions).

**Theorem 1.** Let \( P \equiv (V, I, T, G, A) \) be a planning instance and \( P' \equiv (V', I, T', G', A') \) be its translation. \( P \) has a strong plan \( \pi \) if and only if \( P' \) has a temporal plan \( \sigma \).

**Proof.** (Sketch) Let \( \pi \) be a strong plan for \( P \). Let \( \sigma \) be the plan starting \( a' \) at time \( t \) for each \( a \) starting at time \( t \) in \( \pi \). \( \sigma \) is a valid temporal plan for \( R \) because:

- It achieves the goal \( G' \) of \( R \); all original goals in \( G \) are achieved by \( \pi \) and by \( \sigma \) in the same way, and the goals on the shadow variables must be achieved because \( \pi \) is a strong plan. Given that \( \pi \) achieves the goals regardless of the concrete durations of the actions, it achieves them outside of the uncertainty intervals, where the variables and the shadow variables are aligned.

- Each action \( a' \) is executable in \( R \), because each \( a \) in \( \pi \) is executable in \( P \) regardless of the action durations. Thus the possible uncertainty introduced by the durations is irrelevant for the executability of \( a \) (all the conditions are satisfied). In the translated instance \( R \), all the conditions are also satisfied because the conditions are imposed via the \( \chi \) function that only checks that both the variable and its shadow fulfill the original condition.

- No conflicting effects are possible: the conditions added in \( C_{\mu'} \) prevents any modification of the interested shadow variables during the uncertainty intervals.

Similarly, let \( \sigma \) be a plan for \( R \). Let \( \pi \) be the plan for \( P \) starting \( a \) at time \( t \) for each \( a' \) starting at time \( t \) in \( \sigma \). \( \pi \) is a valid strong temporal plan for \( P \) because:

- It achieves the goal \( G' \), because \( \sigma \) achieves the goal \( G' \) that is a super-set of \( G \) and each translated action has all effects of the original actions.

- Each action \( a \) is executable in \( P \) regardless of the action duration, because each \( a' \in \sigma \) is executable in \( R \) and the conditions in the translated actions are a super-set of the ones in the original action, due to the \( \chi \) function.

- No conflicting effects are possible regardless of the uncertainty duration, because each effect at time \( t \) can be uncertain only between \( \lambda(t) \) and \( \nu(t) \) and we guarantee no other effect is possible in that interval by means of \( C_{\tau'} \).

\[ \square \]

The compilation produces a problem that has: (i) at most twice the number of variables of the original problem, (ii) at most twice the initial and timed assignments and (iii) exactly the same number of actions. The only point in which the compilation might produce exponentially large formulae is in the application of the \( \chi \) function, which is exponential in the number of disjuncts constraining variables appearing in \( L \). Since this only happens for disjunctive conditions, and the number of disjuncts is typically small, this is normally not a serious issue.

5 **Implementation and Experiments**

We conducted two sets of experiments. In the first, we compare our approach against the techniques proposed in (Cimatti, Micheli, and Roveri 2015). This is the only domain-independent planner that we are aware of that can find strong plans for PDDL 2.1 problems with uncontrollable durations. For this experiment, we use an extension to PDDL 2.1 that includes actions with uncontrollable durations (but none of the other extensions that we described in Section 3 such as preconditions and effects at arbitrary times, multi-valued variables, timed-initial-literals, or disjunctive preconditions).

In the second set of experiments, we show the applicability of our technique on a very expressive fragment of the ANML language (Smith, Frank, and Cushing 2008) extended with uncertainty in action durations. Except for action duration uncertainty, ANML natively supports all the other features described in Section 3.

**PDDL with duration uncertainty.** Cimatti, Micheli, and Roveri (2015) extended the COLIN planner (Coles et al. 2012) to solve SPPTUs by replacing the STN scheduler with a solver for strong controllability of STNUs. This yields a sound but incomplete SPPTU planner. The authors
then proposed two techniques to overcome the incompleteness: (1) “Last Achiever Deordering” (LAD) is a sound-but-incomplete technique that tries to limit the incompleteness by building the partial orderings in STNU using the last achiever for each condition that needs support; (2) “Disjunctive Reordering” (DR) is a sound-and-complete technique obtained by considering, at each step, all the possible valid action reorderings using a disjunctive form of STNU.

We compare against this approach by first compiling away temporal uncertainties and then using both the COLIN and POPF planners to solve the compiled instances. We compared our sound and complete technique against both the complete DR and the incomplete LAD. We used a timeout of 600 seconds, a memory limit of 8 GB and the full benchmark set of 563 problems described in (Cimatti, Micheli, and Roveri 2015).

The left plot of Figure 4 reports the cumulative time of the three techniques and the “Virtual Best” solver, obtained by picking the best solving technique for each instance. The right scatter-plot compares our technique (instantiated with COLIN) with the DR approach. The left plot shows that the compilation technique cannot solve as many instances as DR or LAD. However, we note that the “Virtual Best” solver solves many more problems than both the DR and LAD. This shows that the techniques are complementary: problem instances that cannot be solved by LAD or DR are solved quickly by our compilation, and vice-versa. This situation is also visible in the scatter plot: there is a clear subdivision of the problem instances solved by these two different planners.

Our investigation indicates that the main factor that hinders the performance of our approach is the “clip-action” construction (Fox, Long, and Halsey 2004) that is needed to reduce our compilation to PDDL 2.1. In particular, our compilation generates actions with conditions and effects that occur at intermediate times. Compiling this to PDDL 2.1 requires three PDDL 2.1 actions for each action in $A_r$: a container action, and two actions inside the container action that are clipped together. This deepens the search and lengthens the plans for COLIN and POPF.

**ANML with duration uncertainty.** As described in Sections 3 and 4, our framework handles many useful features beyond PDDL 2.1. Some of these can be represented in higher levels of PDDL (e.g., multi-valued variables), some cannot (e.g., arbitrary timed action conditions and effects). While comparing against current state-of-the-art in PDDL 2.1 shows the feasibility of our approach, it restricts us to a subset of features that can be handled by our compilation. Moreover, as discussed above, the limitation of PDDL 2.1 adversely impacts the performance of our approach.

To show the full expressive potential of our approach, we used the Action Notation Modeling Language (ANML) (Smith, Frank, and Cushing 2008), which can natively model all these constraints. ANML is a variable-value language that allows high-level modeling, complex conditions and effects, HTN decomposition and timeline-like temporal constraints. Our only addition to ANML is the capability to model uncertain action durations: `duration :in [l, u]` where $l$ and $u$ are constant values specifying the lower and upper bounds on the duration of $a$. We name our

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4Our approach allows the use of any PDDL2.1 planner that can handle required concurrency. Unfortunately, many temporal planners such as LPG and TemporalFastDownward do not support this, and therefore cannot find solutions to the problems generated by our compilation.
Model simplification: it is sometimes possible to simplify a strong planning problem with temporal uncertainty by considering the maximal or minimal duration of an action having uncertain duration. As we discussed in Section 3, this "worst-case" approach is in general unsound; nonetheless, it is possible to recognize some special cases in which it is sound and complete. This simplification can be done upfront and could be beneficial for both our compilation and the approaches in (Cimatti, Micheli, and Roveri 2015). Precisely understanding when and how this simplification can be applied is an open problem, but a preliminary analysis suggests that an action $a$ with uncertain duration $[l, u]$, having conditions involving variables in $V_C$ and effects involving variables in $V_E$, can be considered as being controllable in $[u, u]$ (without changing its conditions or effects) if all the following conditions hold:

- $A$ is mutually exclusive with any other action that has an effect on a variable in $V_E \cup V_C$.
- No Timed Initial Literal modifies a variable in $V_E \cup V_C$.
- Every action with a condition involving $V_C$ is mutually exclusive with $A$.

These strict requirements are sufficient to guarantee that the action can be considered to last for its maximal duration as it is impossible to impose a "lower-bound" constraint in any valid execution of the actions.

Increase expressiveness: Even though the formalization we presented is quite expressive and general, the ANML language has many features that are not covered. A prominent example is the support of conditional effects, which cannot be expressed in our language but are possible in both ANML and PDDL. We note that, analogously to disjunctive preconditions, the common compilation of conditional effects is unsound in the presence of temporal uncertainty, because it transforms a possibly uncontrollable effect into a controllable decision for the planner. Nonetheless, our translation (with some modifications) is still applicable in the presence of conditional effects, but it sacrifices completeness. The intuitive reason is that we represent uncertainty as a pair of variables (original and shadow) with the assumption that the value of the variable in the original execution is either of the two values. With conditional effects, it is possible to design models in which the variable can actually be uncertain between more than two values.

Improve performance: Finally, we would like to study ways to overcome the disappointing performance of the compilation into PDDL by hybridizing the “native” DR and LAD techniques with our approach to exploit their complementarity. Another possibility is to modify a temporal planner so that it understands the clip-action construct and avoids useless search when dealing with the instances produced by our translation.

### 6 Future Work

While the preliminary results are promising, we are considering several possible extensions.
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References


Cimatti, A.; Micheli, A.; and Roveri, M. 2015. Strong temporal planning with uncontrollable durations: a state-space approach. In AAAI.


